Homework 1

1. EX.7 P.14

Since & , then

And & , then

Therefore, by definition

Similarly…

Since & , then

And & , then

Therefore, by definition

1. EX. 19 P.21

Let’s assume the contrary, that is that . We know is a subspace of . Let , then .

If , then , which (based on Theorem 1.3 (b))

And similarly, if , then , which

Since both lead to a contradiction this implies that our original assumption was incorrect and that it follows that

Since we know that, , then . In either case, is a subspace of , by the fact that are subspaces of .

1. Ex. 28 p.23

As Theorem 1.3 condition hold, we know that of all matrices (including skew symmetric) are a subspace of .

We also know that the zero matrix is equal to its transpose and thus belongs to . Based on previous examples, it is easily proven that for any matrices and any scalars , .

If we say , then it follows that and . Therefore, , which then means .

Finally, , then , then for any , we know , therefore,

1. Ex. 29 p.23

Because we know are closed under addition and scalar multiplication, it follows that . We know, and similar to Q28, are both subspaces of , contains matrix , a lower triangular matrix. Therefore, .

1. EX. 3 P.41

We choose and

Therefore, it is linearly dependent

1. EX. 9 P.55

Let . We can compute,

Then,

We then solve for,

Therefore, we conclude,

1. EX. 17 P.56

By definition, a skew-symmetric matrix is , in terms of entries in the matrix this means .

The basis of a matrix is , therefore, the basis for skew-symmetric matrix is .

By definition we know, the vector space has dimension , thus we know that a vector space has dimension ( – both identical sides of the diagonal, the diagonal itself).

It then follows that a skew-symmetric vector space has the dimension , ( to remove the diagonal, which is all zeroes).

1. EX. 10 P.75

From Example 12, we know that can be defined by

Therefore, we can see that so is one-to-one.

1. EX. 3 P.84

1. EX.4 P.84